Fast Multipole Method for complex flows

Hassib Selmi[1], Giovanni Ghigliotti[2], Chaouqi Misbah[3], Lassaad El Asmi[1]

[1] Laboratoire d'Ingénierie Mathématique, Ecole Polytechnique de Tunisie
B.P. 743 - 2078 La Marsa, TUNISIE.
[2] Dept. Chemical and Biological Engineering University of British Columbia Vancouver, BC V6T 1Z3, CANADA.
[3] Univ. Grenoble 1 / CNRS, LIPhy UMR 5588, Grenoble, F-38041, FRANCE.

We first address the problem of numerical simulations of a series of vesicles in an unbounded (we disregard here the wall effects) Poiseuille (parabolic) flow in two dimensions . A vesicle is a simple model that mimics dynamics of red blood cell (RBC): they are drops of fluid surrounded by a locally inextensible membrane, endowed with a bending energy. The understanding of their behaviour in a parabolic velocity profile is essential in order to answer several important questions such as rheology, spatio-temporal organisation of RBCs (and so on) in the blood circulatory system.

At the scale of a RBC the Reynolds number is small in different blood vessels of the microvasculature, such as capillaries (but not necessarily in arterioles) so that the Stokes approximation is legitimate in this regime. Despite the linearity of the Stokes equations, the equations of the evolution of the shapes of RBCs are nonlinear (due to the free boundary character). Taking advantage of the linearity of the Stokes equations, we make use of the Green's function techniques that allows one to convert the problem into a boundary integral one (usually called BEM –Boundary Element Methods).

One major difficulty regarding numerical simulations in BEM arises from the fact that the problem is nonlocal. This non-locality means that the complexity is $O(N^2)$ (where N is the number of discretization points on the membranes of RBCs). In order to overcome this difficulty we develop a fast multipole method (FMM) to minimize the computing time of the product matrix-vector as well as the memory space reserved for the matrix storage by lowering the two costs from $O(N^2)$ to $O(N)$

We then consider another problem, namely dynamics and rheology of vesicles in a couette geometry, where the boundaries of the cylinders are taken into account. The boundaries induces new numerical questions: the determination of the stress due to the walls, which amounts to inverting large matrices. This problem is solved by means of GMRES method (Generalized Minimal Residual Method) coupled with FMM. We illustrate our study by presenting several results of simulation of a large number of vesicles.

References

[1] Hassib Selmi, Giovanni Ghigliotti, Chaouqi Misbah and Lassaad El Asmi, *Boundary integral and fast multipole method for two dimensional vesicle sets in Poiseuille flow*, AIMS Journal, Discrete and Continuous Dynamical Systems-B, **15-4** (2011), 1065–1076.

[2] Giovanni Ghigliotti, Thierry Biben and Chaouqi Misbah, *Rheology of a dilute two-dimensional suspension of vesicles*, Journal of Fluid Mechanics, **653** (2010), 489–518.

[3] Giovanni Ghigliotti, Hassib Selmi, Badr Kaoui, George Biros and Chaouqi Misbah, *Dynamics and Rheology of highly deflated vesicles*, ESAIMproc, **28** (2009), 211–226.

[4] Hassib selmi, Hatem Hamda and Lassaad El Asmi *Fast multipole method for the 3D Stokes flow: truncation error analysis ans asymptotic complexity*, (International conference on Numerical analysis and applied mathematics, corfu, Greece 16-20 September 2007) Americain Institute of Physics AIP conf Proceedings, **936** (2007), 499–502.

[5] N.A. Gumerov and R. Duraiswami *Fast multipole method for the biharmonic equation*, J. Comput. Phys, **215(1)** (2006), 363–383.

[6] L. Greengard and V. Rokhlin *A fast algorithm for particle simulations*, J. Comput. Phys, **73** (1987), 325–348.